Une école de l'IMT

From compressive sensing to analog to information converters

Paul Chollet
pachollet@telecom-paristech.fr

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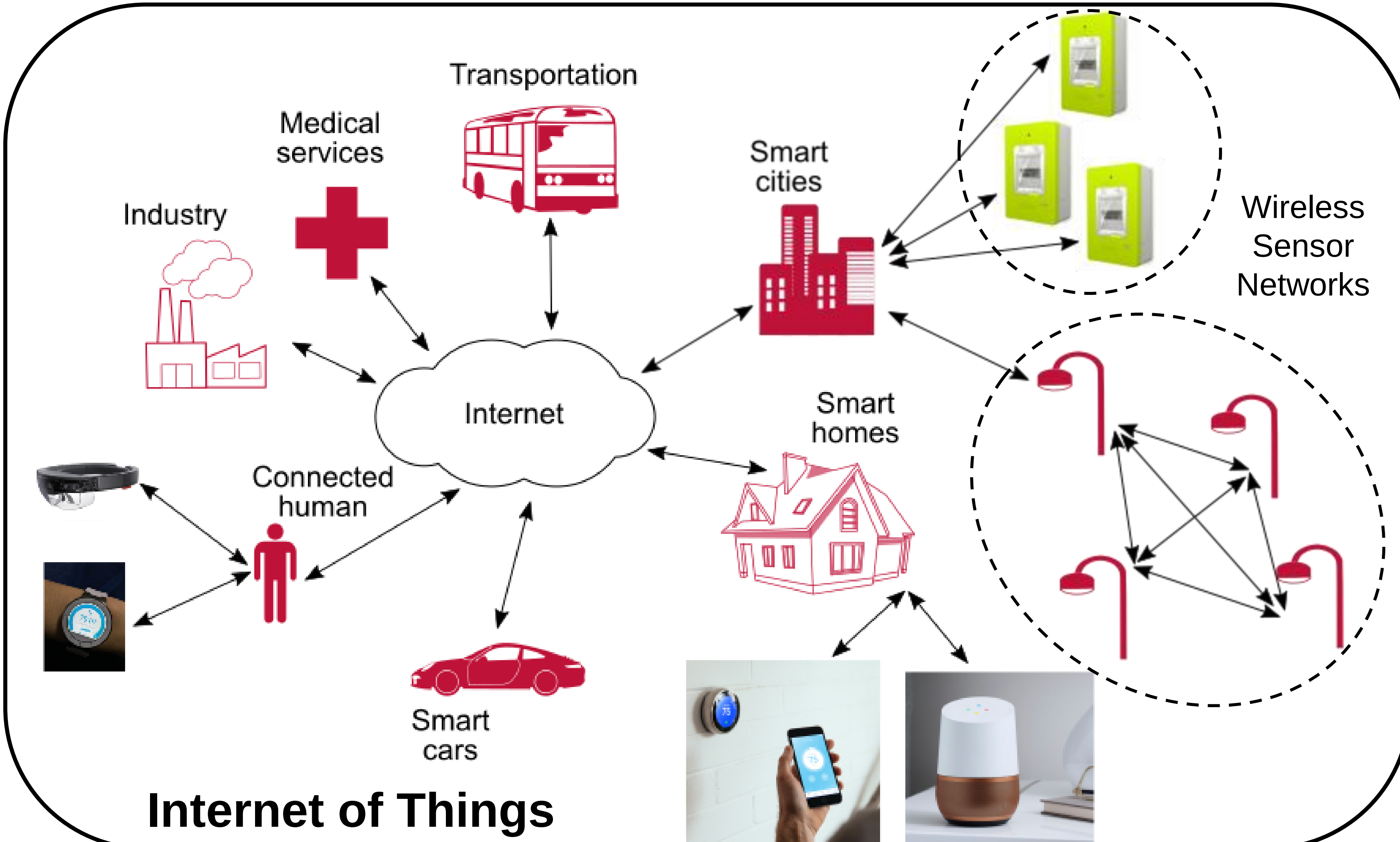
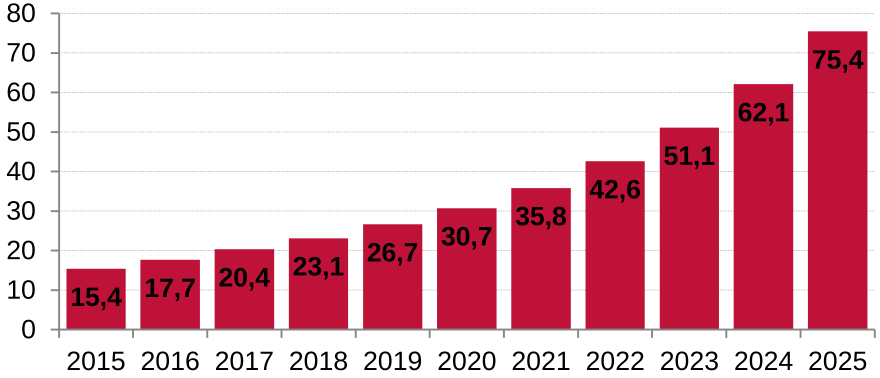
The Internet of Things (IoT)



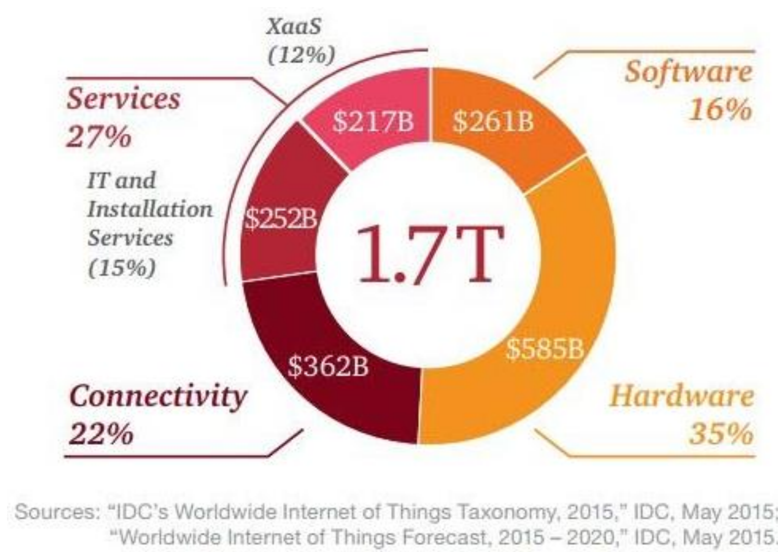
Global IoT market predictions

Connected devices in billions

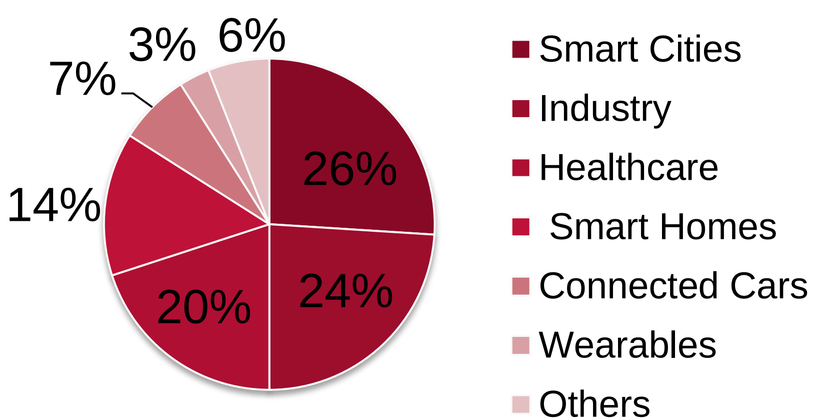


Source: Statista 2015

IoT revenue by technology in 2020



IoT by sector



Source: Growthenabler 2017

Digital data volume explosion

Presentation summary

1. Wireless sensor networks and smart sensors
2. Principle of compressive sensing
3. Architecture for analog to information converters
4. Signal reconstruction algorithms
5. Conclusion

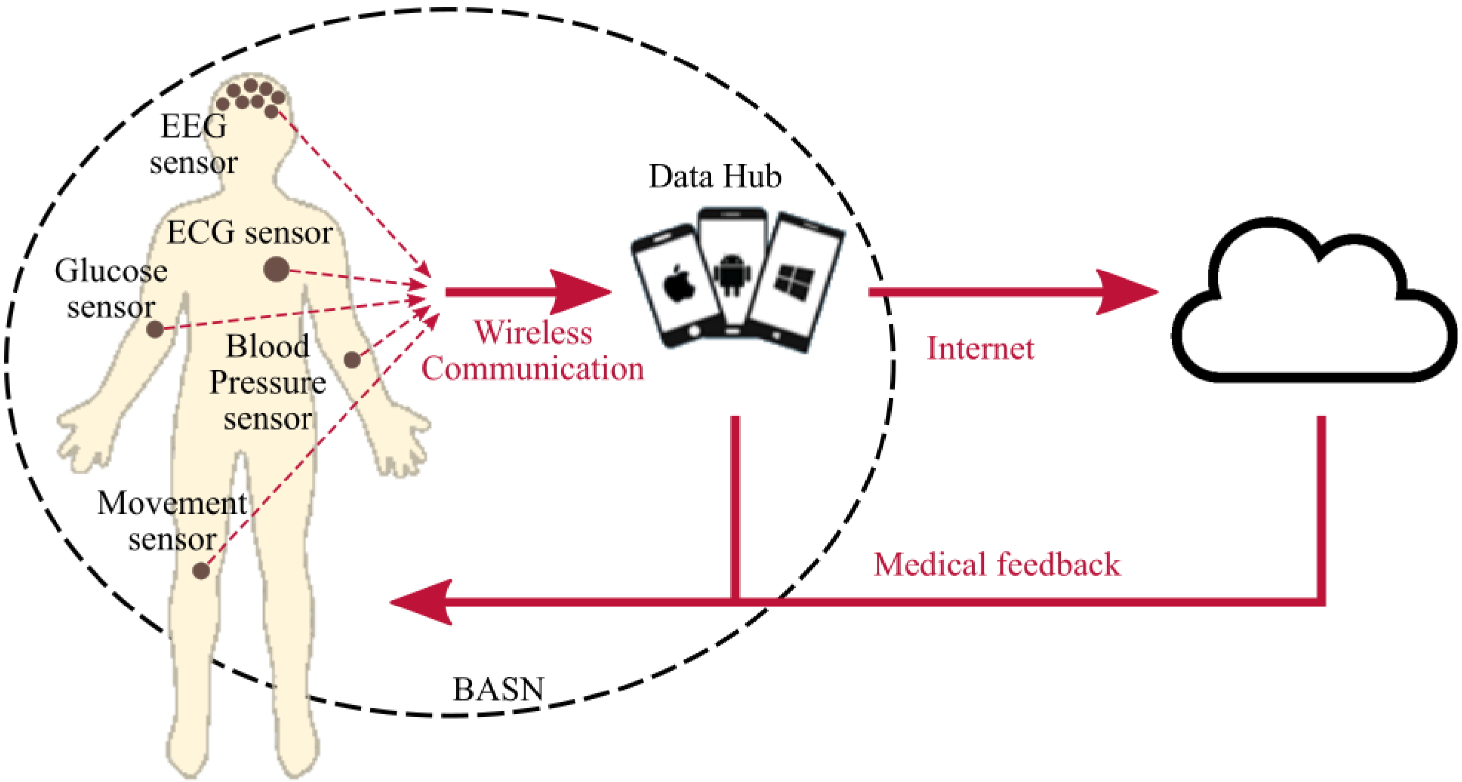
1. **Wireless sensor networks and smart sensors**
2. Principle of compressive sensing
3. Architecture for analog to information converters
4. Signal reconstruction algorithms
5. Conclusion

Wireless sensor networks and smart sensors

- What are the constraints for WSN and smart sensors ?
- How to overcome these constraints ?

Wireless sensor network architecture

Example of the Body Area Sensor Network



Group activity:

- 4/5 students, 5 minutes
- Make a list of the constraints for the sensors and rank them

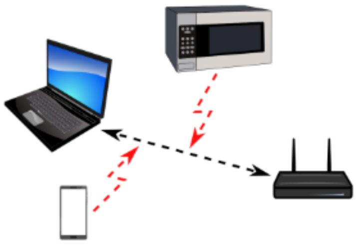
New challenges

Security

- Data confidentiality and integrity
- Security mechanisms embedded into circuits

Interoperability

- Interferences between WSN
- Specific communication protocols



Data transfer

- Increasing amount of data and bandwidth
- Saturation of the RF spectrum

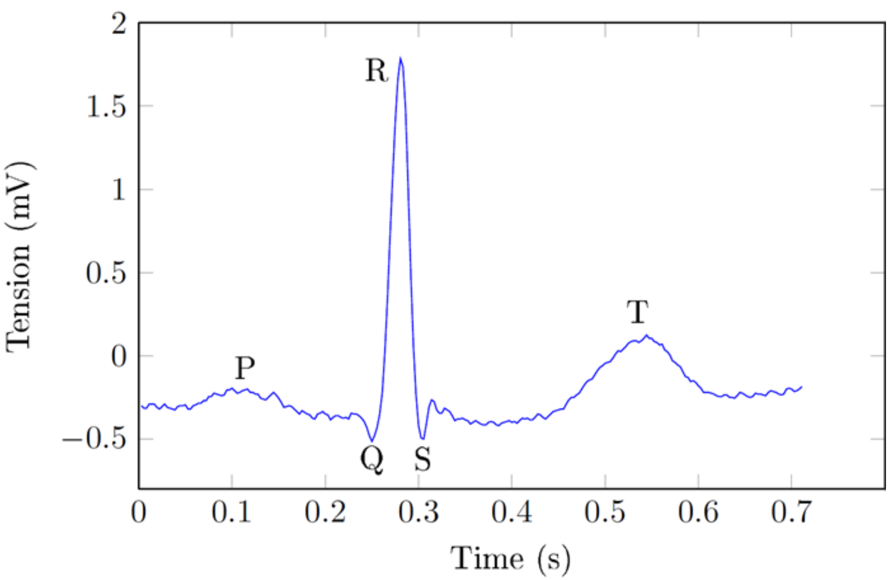
Power consumption

- Sensor working on batteries
- Energy source replacement is not always possible

Example 1: electrocardiogram (ECG) signal

Cardiac arrhythmia detection

- Heart diseases responsible for 15.5 % of worldwide death
- Well studied subject



ECG signal characteristics

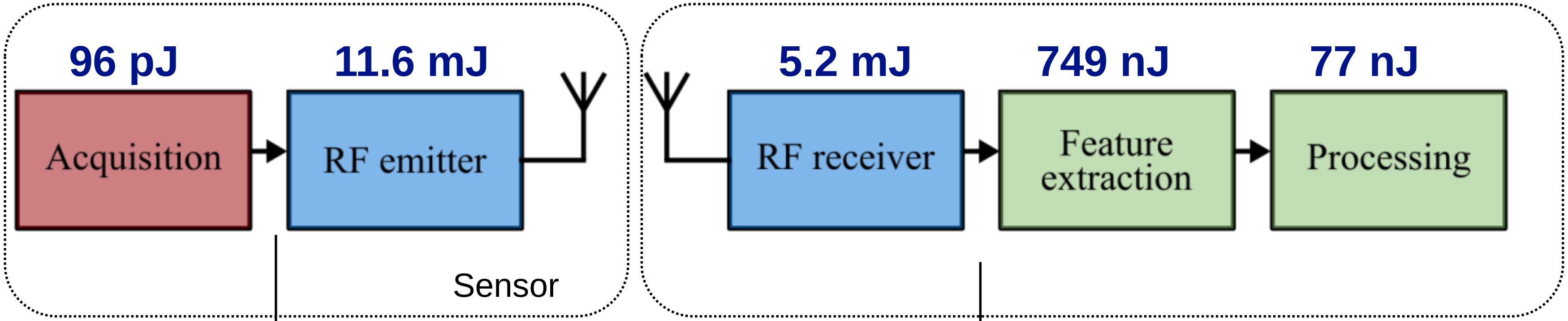
- Continuous signal
- Cycle duration: 0.5 – 0.9 s
- Sampling frequency: 200 – 1000 Hz
- Precision: ~ 10 bits

Application

- Arrhythmia detection from [1]
- Signal is 800 10-bit samples
- 1 kHz sampling frequency

[1] T. Chen *et al* . Design of a Low-Power On-Body ECG Classifier for Remote Cardiovascular Monitoring Systems. IEEE Journal on Emerging and Selected Topics in Circuits and Systems, March 2013

Example 1: simple sensor



Analog-to-digital converter [2]

- 0.18 μm CMOS
- 0.12 pJ/sample

Bluetooth Low Energy transceiver [3]

- 0.13 μm CMOS
- Emitter: 14.5 mW
- Receiver: 6.5 mW

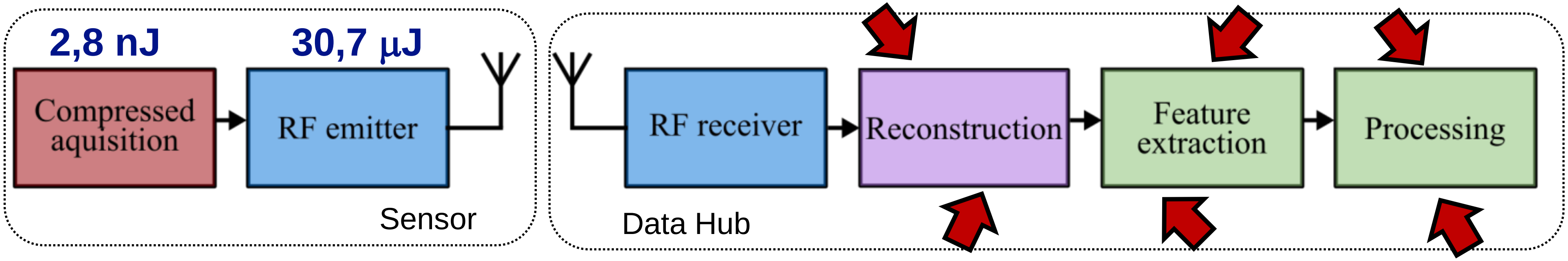
Arrhythmia detection with linear classification [1]

- 0.13 μm CMOS
- Post-layout simulation

Transmission require the most energy

[2] L. Yan *et al*. A 0.5- V 12- W Wirelessly Powered Patch-Type Healthcare Sensor for Wearable Body Sensor Network. IEEE Journal of Solid-State Circuits, November 2010.
[3] A. C. W. Wong *et al*. A 1 V 5 mA Multimode IEEE 802.15.6/Bluetooth Low-Energy WBAN Transceiver for Biotelemetry Applications. IEEE Journal of Solid-State Circuits, January 2013.

Example 1: using compression



Compress the data during acquisition: compressed sensing

- Use knowledge on signal structure
- Reduce the amount of data to be transmitted

Analog-to-information converter [4]

- 0.13 µm CMOS
- 4 fold compression
- 14 pJ/compressed sample

Sensor energy requirement is divided by 377

Limitations

- Reconstruction Algorithm is complex
- Reconstruction error increases with the compression factor

[4] D. Gangopadhyay *et al*. Compressed Sensing Analog Front-End for Bio-Sensor Applications. IEEE Journal of Solid-State Circuits February 2014.

Example 2: Astrophysical measurements

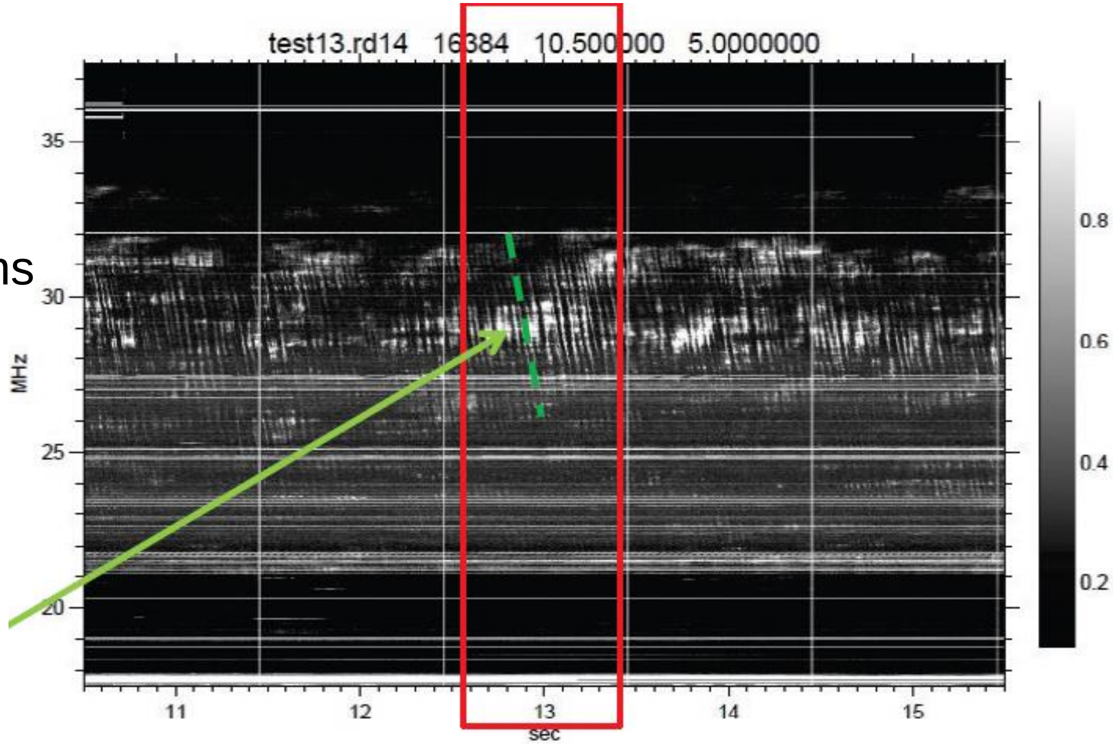
Radio emission from Jupiter

- S-burst: interaction between Jupiter and its satellite Io
- Range: [0.45 – 40] MHz
- Digital receiver connected to the Nançay Decametric Array in France

Example 2: Astrophysical measurements

Receiver

- Spectrogram of Jovian Signal
- 14 bit @ 80 Ms/s
- 5 sec = 5.6 billions bits
- Vertical lines: periodic calibrations
- Horizontal lines: terrestrial radio broadcasts (radars, radio, TV)



Useful information

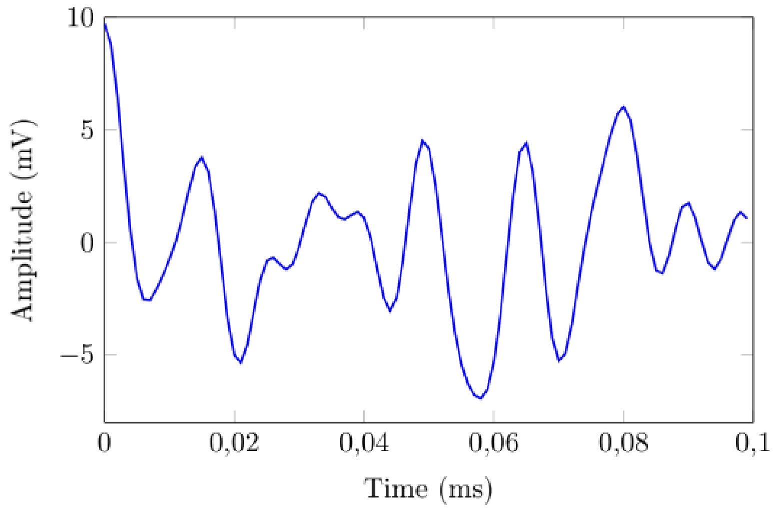
- Frequency drift: green slope
- Value around 20 MHz/s

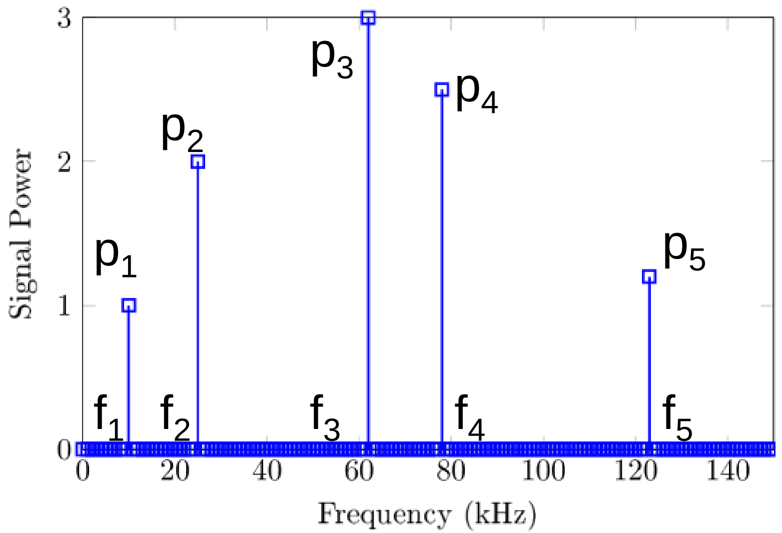
How to get this information with far fewer samples?

Principle of compressive sensing

- What is compressive sensing?
- What are its pros and cons?

Nyquist and Information rate





Nyquist Rate

- Shannon-Nyquist theorem for perfect reconstruction: $F_s \geq 2F_{max} = 2B$
- Signal representation require 2*W*T samples
- For T=1 ms and B=123 kHz => 246 samples

Information Rate

- Entropy of the signal
- Reduced by a-priori information
- 5-tone signal: 10 elements
  - f_1 to f_5 and p_1 to p_5

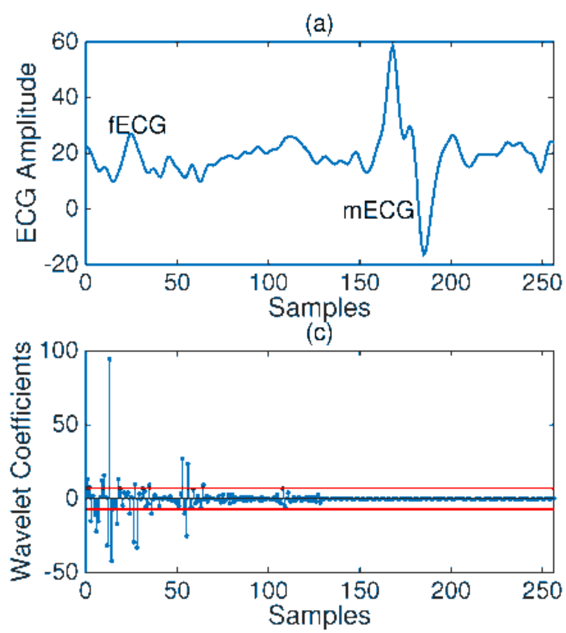
CS: Acquire signal as close to IR as possible

Sparse signal

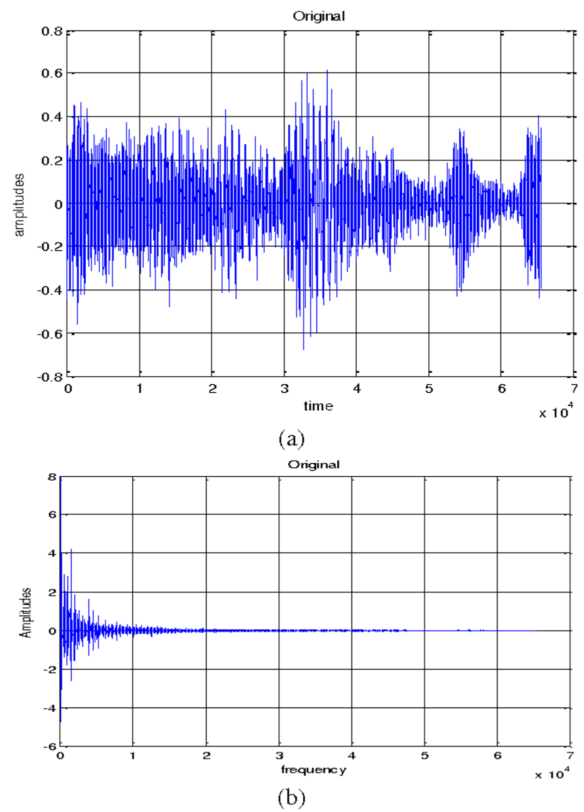
Definition

A signal is k-sparse if it can be represented with only k non-zero element in a specific base

Example



ECG signal and its 4-level Db4 wavelet transform



Audio signal and its Fourier transform

Acquisition of sparse signal

Signal x in time domain

Sparse signal s
k-non zero entries

$N \times 1$ $\quad N \times N$ $\quad N \times 1$

$$\boldsymbol{x} = \boldsymbol{\Psi s}$$

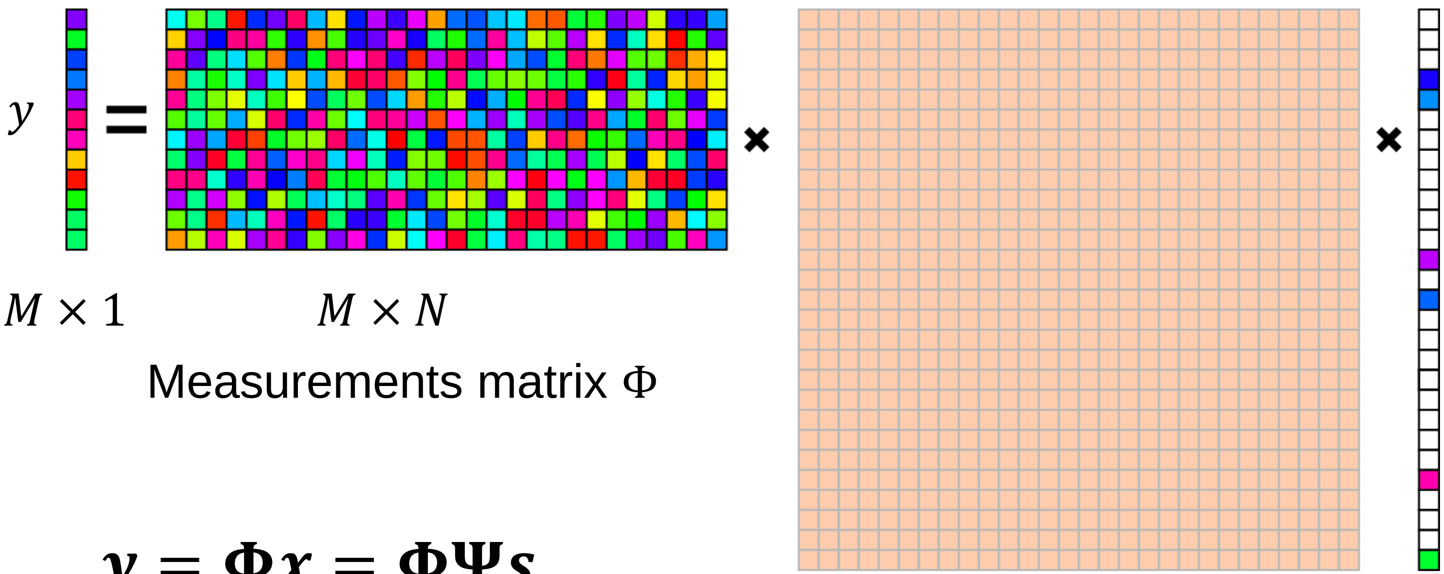
Typical sparsity basis

- Time: $\Psi = I_N$
- Fourier domain: $\Psi = DFT^{-1}$
- Wavelet domain: $\Psi = DWT^{-1}$

Compressed sensing

Principle

- Take only M samples or linear measurement instead of N



$M \times 1$ $\quad$ $M \times N$ $\quad$ $N \times N$ $\quad$ $N \times 1$

Measurements matrix Φ

$$\boldsymbol{y} = \boldsymbol{\Phi x} = \boldsymbol{\Phi \Psi s}$$

$$M \ll N$$

Can we find x or s from y ?

Signal Recovery

Principle

- Given $y = \Phi\Psi s$
- Find s with maximal sparsity
- $M \ll N$: Many possible solutions ill-posed inverse problem

Solution: add constraint

- Use $l_2$norm: $\hat{s} = \arg\min\|s\|_2 \; s.t: \; y = \Phi\Psi s$
  - Wrong: output a non sparse solution

- Use l_0 norm: $\hat{s} = \arg\min\|s\|_0 \; s.t: \; y = \Phi\Psi s$
  - Exact solution but NP hard problem (require exhaustive search)

- Use l_1 norm: $\hat{s} = \arg\min\|s\|_1 \; s.t.: \; y = \Phi\Psi s$
  - Linear problem and high probability of exact solution if M big enough [5]
  - Minimum M depends on measurement matrix and signal sparsity

True for any measurement matrix Φ ?

[5] E. J. Candes, J. Romberg, and T. Tao. Robust uncertainty principles : exact signal reconstruction from highly incomplete frequency information. *IEEE Transactions on Information Theory*, 52(2) :489–509, February 2006.

Signal Recovery Condition

Restricted Isometry Property (RIP)

- Assuming s is k-sparse
- $\forall p_k\ k-sparse\ (1-\delta_k)\|p_k\|_2^2 \leq \|\Phi\Psi p_k\|_2^2 \leq (1+\delta_k)\|\mathrm{p}_k\|_2^2$
- $0 \leq \delta_k \leq 1$ is the isometry constant of $\Phi\Psi$

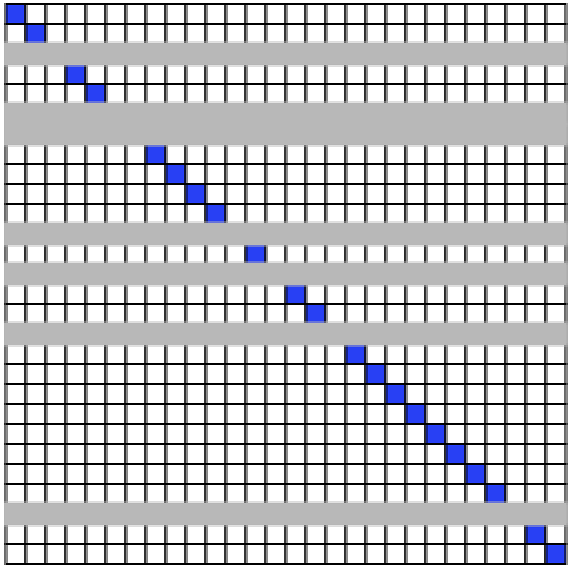
Hard to check in practice

Coherence

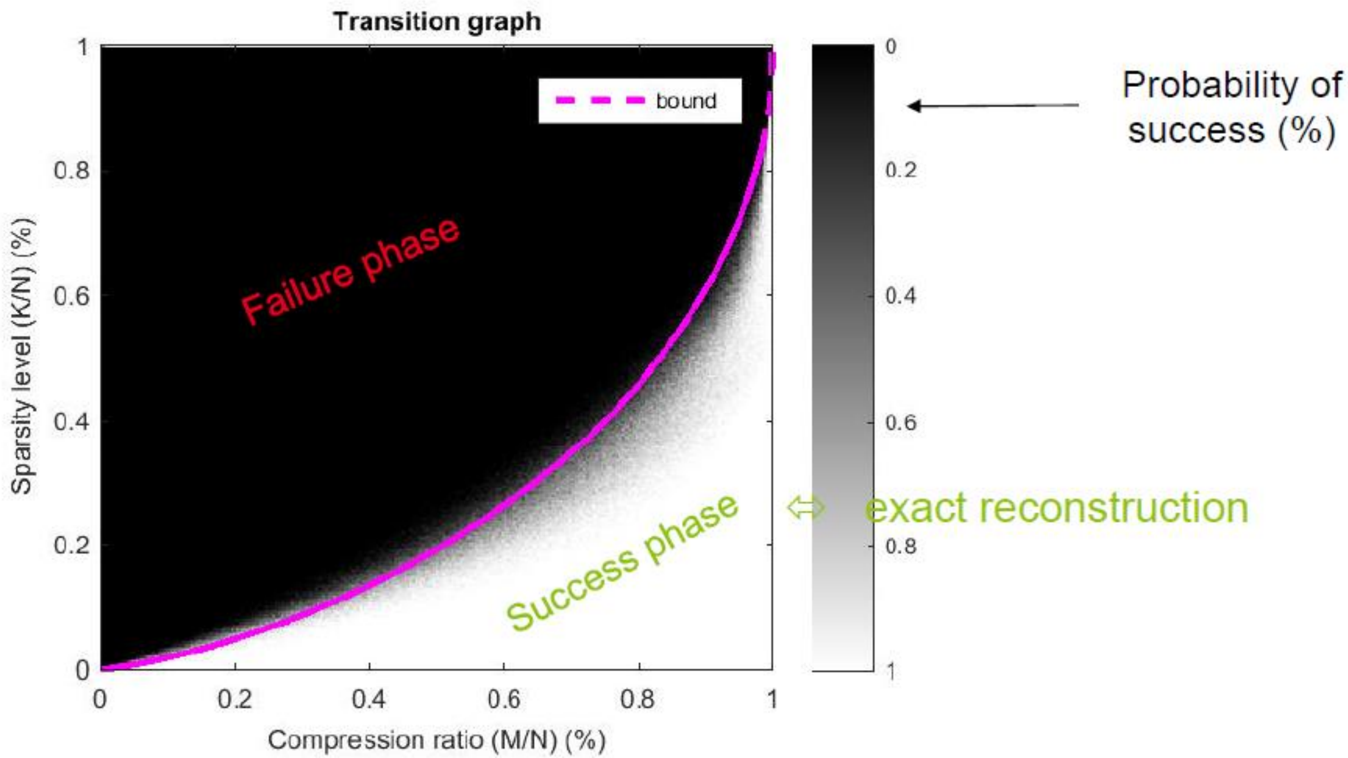
- $\mu(\Phi,\Psi) = \sqrt{N} \max_{1\leq j,k\leq N} |\langle \phi_j, \psi_k^T \rangle|$
- Ensure that every measurement carries useful amount of information of non-zero element of s.
- Must be as close as possible to lower bound 1

Practical measurement matrices

- Random matrices
- +/- 1 Bernoulli matrices
- Random row selection matrices (Only for non time sparse signal)



Performance evaluation



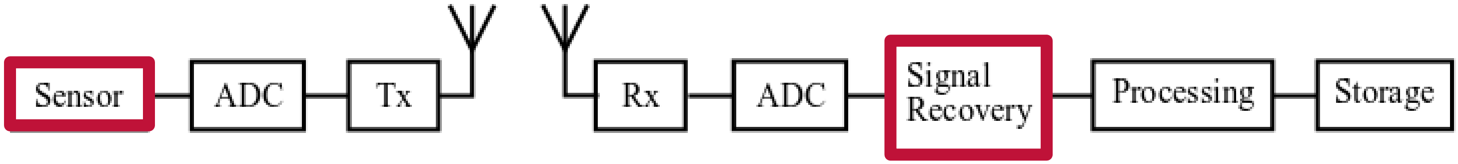
Reconstruction SNR (RSNR)

$$RSNR = \frac{\|s\|_2^2}{E(\|\hat{s} - s\|_2^2)}$$

Where to apply CS in processing chain?

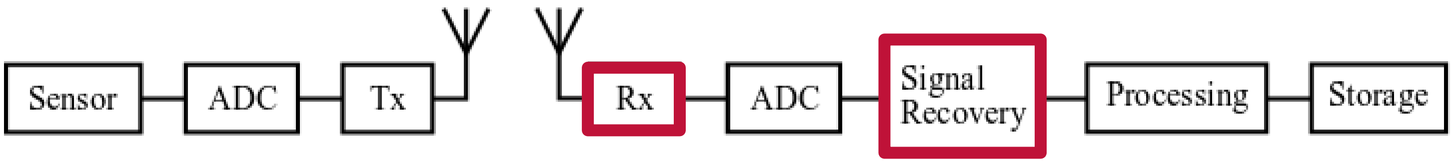
Sensor level

- Relax RF data throughput



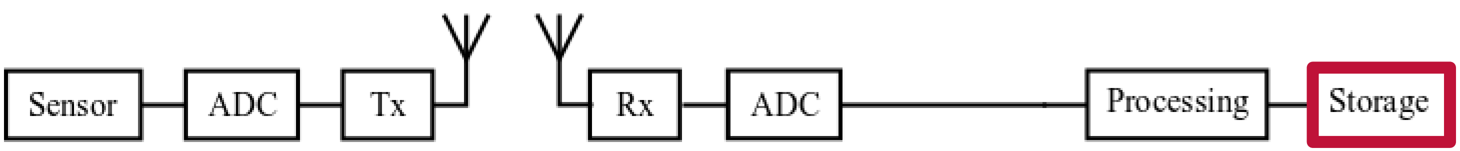
Analog Receiver level

- Relax RF signal acquisition



Storage level (compression)

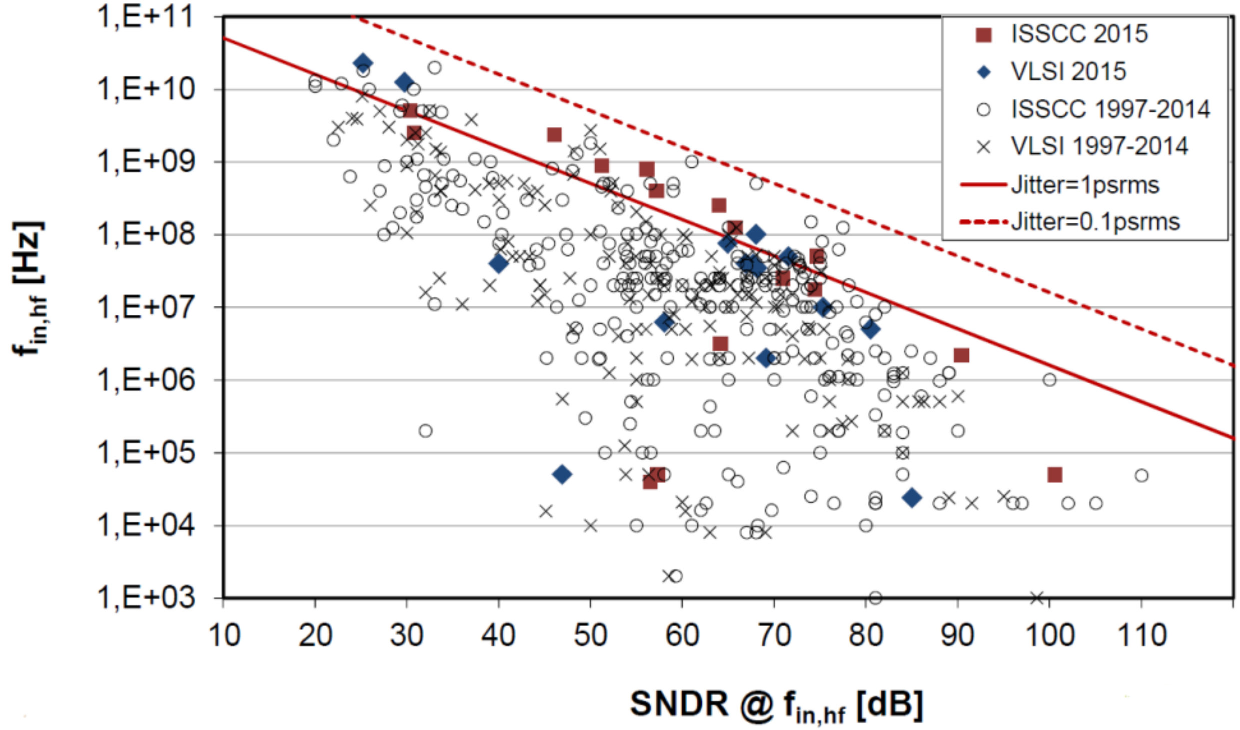
- Relax memory requirements



Architecture for analog to information converters

A2I converter advantages

Traditional ADC are limited in the maximum achievable SNDR for a given bandwidth



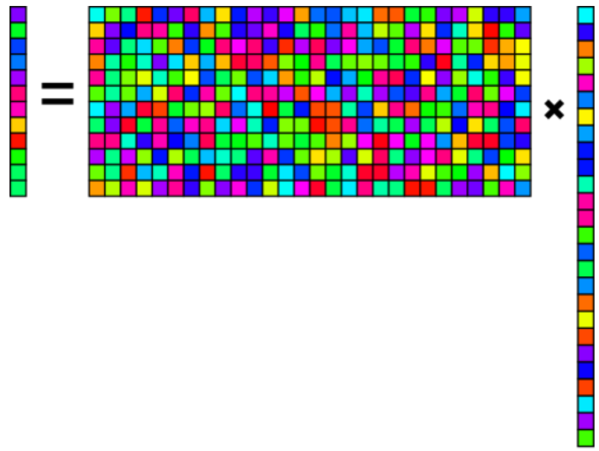
(Murmann 2015)

Objectives

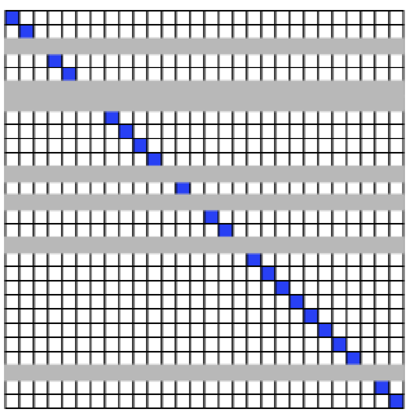
- Improve effective bandwidth by using sparsity of input signal
- Improve resolution (SNDR) by sub-Nyquist sampling
- Decrease the amount of sample acquired

Let’s think a bit

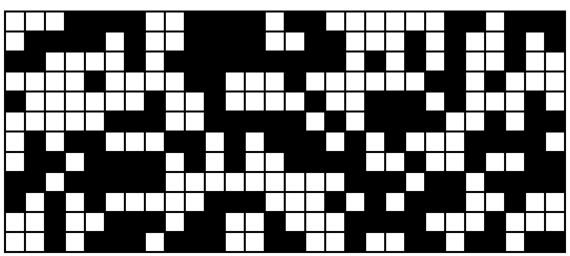
- Form three team within the class
- Each team think of an architecture to implement a A2I converter



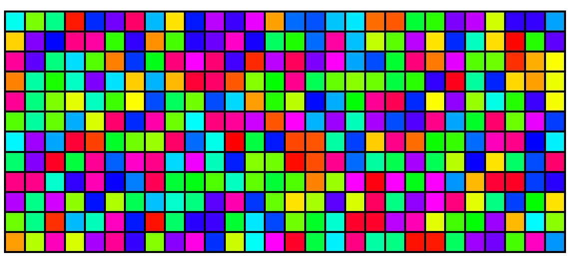
Team 1



Team 2

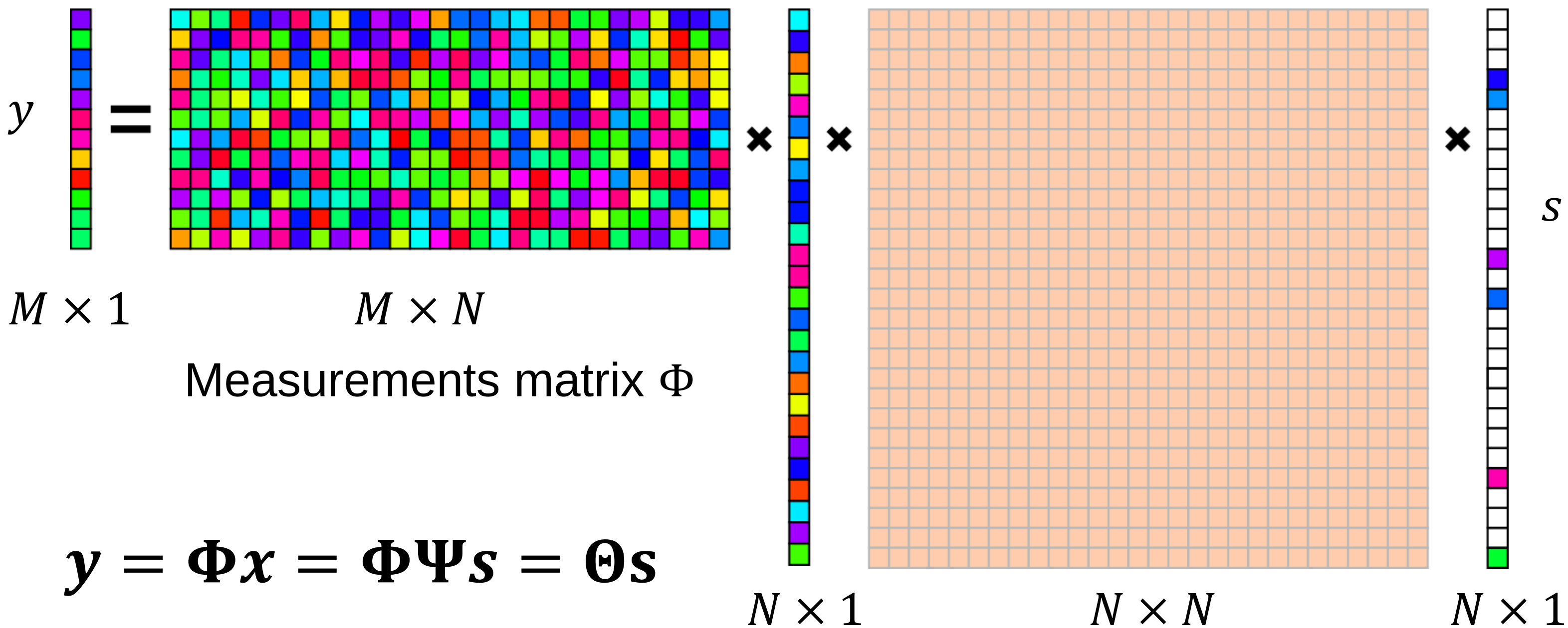


Team 3



Signal reconstruction algorithms

Problem statement



$$\boldsymbol{y} = \boldsymbol{\Phi x} = \boldsymbol{\Phi \Psi s} = \boldsymbol{\Theta s}$$

$$M \ll N$$

Can we find s from y ?

Convex optimization solution

Basis Pursuit (BP)

- Solve: $\hat{s} = \arg\min \|s\|_1 \; s.t.: \; y = \Theta s$
- Will find the sparsiest solution
- Might not converge if noisy measurements

Least absolute shrinkage and selection operator (LASSO)

- Solve: $\hat{s} = \arg\min \|s\|_1 \; s.t.: \|y - \Theta s\|_2 < \epsilon$
- The solution is more robust to noise
- Noise controlled with ϵ

Basis Pursuit De-Noising (BPDN)

- Solve: $\hat{s} = \arg\min \|y - \Theta s\|_2^2 + \lambda \|s\|_1$
- Make a compromise between error and sparsity
- Controlled by λ

Solving method

- Primal-dual interior point
- Adaptive gradient

Complexity in $O(N^3)$!

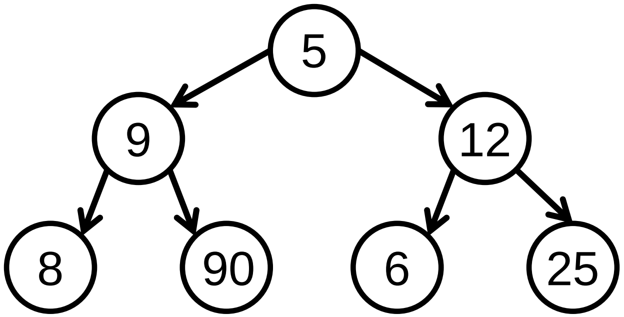
Greedy algorithm

Principle

- Find a solution with iterative process
- At each iteration choose the best solution according to certain criteria
- Pro: less complex
- Con: does not guarantee a global optimum, only a local one

Example

- Find the largest sum while descending a tree
- Greedy criteria: choose the path providing the highest result



Greedy solution: 42
Optimal solution: 104

Greedy algorithm

Orthogonal Matching Pursuit

- Find a K-sparse solution by selecting non-zero element one by one

Input

- $M \times N$ measurement matrix Θ
- Measurement vector y
- K

Initialization

- Residual: $r_0 = y$
- Index set: $\Omega_t = \emptyset$
- Matrix of chosen atom: $\Theta_0 = [\,]$
- Vector of signal amplitude: $a_0 = [\,]$
- Increment index: $t = 1$

Procedure

1. Find: $\omega_t = \arg \max_{1 \leq j \leq N} |\langle r_{t-1}, \theta_j \rangle|$ — Find column with maximum correlation with residue
2. Update: $\Omega_t = \Omega_{t-1} \cup \omega_t$; $\Theta_t = \left[\Theta_0 \;\; \theta_{\omega_t}\right]$
3. Solve: $a_t = \arg \min_x ||\Theta_t x - y||_2$ — Least square problem
4. Update residual: $r_t = y - \Theta_t a_t$
5. Do $t = t + 1$
6. Repeat until $t = K$

Greedy algorithm

Compressive Sampling Matching Pursuit (CoSaMP)

- Similar to OMP
- Select the $2K$ maximum correlation columns
- Keeps only the K highest value of a_t

Normalized Iterative Hard Thresholding (NIHT)

- Do not try select columns to minimize residual
- Select solution:
  - Minimizing the residual
  - Maximizing the difference between the current solution and previous one

Method	OMP	CoSaMP	NIHT
Complexity	$O(KMN)$	$O(\log(K)MN)$	$O(\log(K)MN)$

Algorithms comparison for biosensors [9]

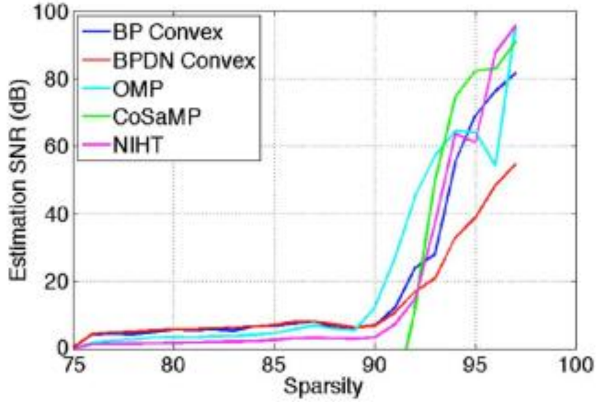


Fig. 15. ECG signal reconstruction accuracy.

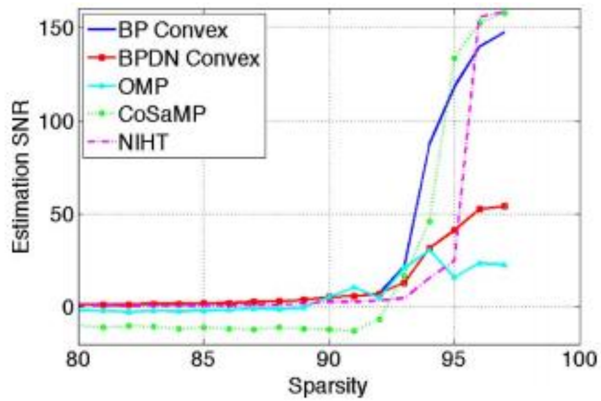


Fig. 17. EMG time-domain signal reconstruction accuracy.

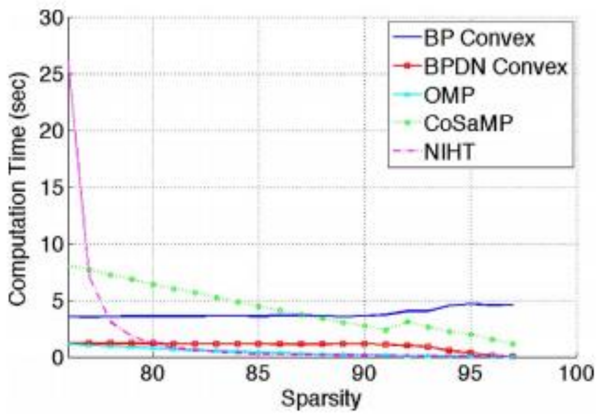


Fig. 16. ECG signal reconstruction computation time.

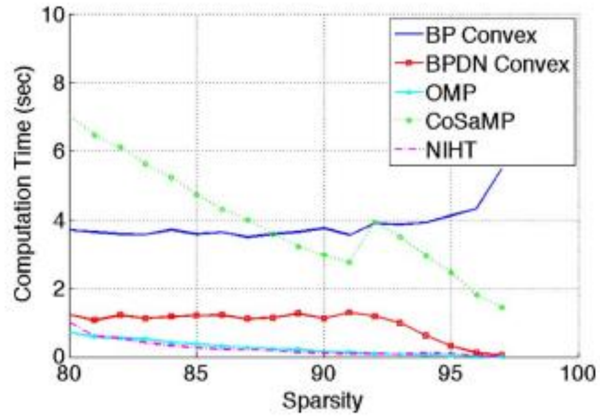


Fig. 18. EMG time-domain signal reconstruction computation time.

There is no general best choice

[9] A. M. R. Dixon, E. G. Allstot, D. Gangopadhyay and D. J. Allstot, "Compressed Sensing System Considerations for ECG and EMG Wireless Biosensors," in *IEEE Transactions on Biomedical Circuits and Systems*, vol. 6, no. 2, pp. 156-166, April 2012.

Conclusion

Conclusion

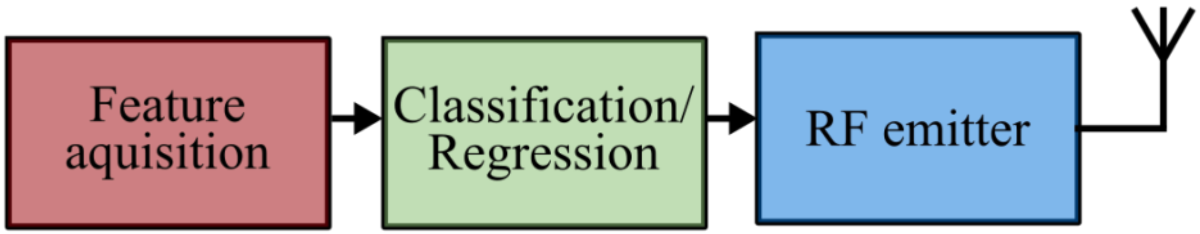
CS take advantages of the signal sparsity to overcome sensing problems

But there are many remaining challenges

- Acquisition and reconstruction must be robust to acquisition noise
- How to quantize the measurement?
- How to find the ideal basis giving a sparse signal
- Use of random matrices. Could we find more practical matrices?
- Development of efficient embeddable recovery algorithms
- Can we exploit the signal in its compressed form?

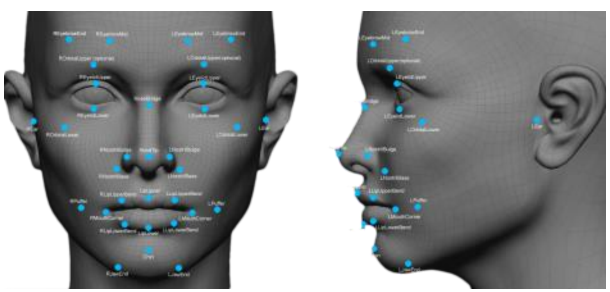
Towards Analog to feature/classification converter

- Extract feature directly from the analog signal
- Use classifier with the extracted features

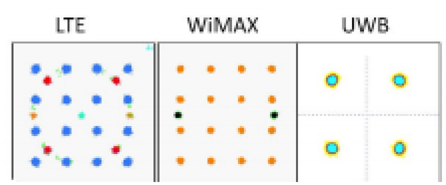


Compressed sensing?

Detect diseases



Facial recognition



RF constellation recognition

Activity detection

References

[1] T. Chen *et al* . Design of a Low-Power On-Body ECG Classifier for Remote Cardiovascular Monitoring Systems. IEEE Journal on Emerging and Selected Topics in Circuits and Systems, March 2013

[2] L. Yan *et al*. A 0.5- V 12- W Wirelessly Powered Patch-Type Healthcare Sensor for Wearable Body Sensor Network. IEEE Journal of Solid-State Circuits, November 2010.

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[5] E. J. Candes, J. Romberg, and T. Tao. Robust uncertainty principles : exact signal reconstruction from highly incomplete frequency information. *IEEE Transactions on Information Theory*, 52(2) :489–509, February 2006.

[6] M. Trakimas, R. D'Angelo, S. Aeron, T. Hancock and S. Sonkusale, "A Compressed Sensing Analog-to-Information Converter With Edge-Triggered SAR ADC Core," in *IEEE Transactions on Circuits and Systems I: Regular Papers*, vol. 60, no. 5, pp. 1135-1148, May 2013.

[7] J. N. Laska, S. Kirolos, M. F. Duarte, T. S. Ragheb, R. G. Baraniuk and Y. Massoud, "Theory and Implementation of an Analog-to-Information Converter using Random Demodulation," *2007 IEEE International Symposium on Circuits and Systems*, New Orleans, LA, 2007, pp. 1959-1962.

[8] F. Chen, A. P. Chandrakasan and V. M. Stojanovic, "Design and Analysis of a Hardware-Efficient Compressed Sensing Architecture for Data Compression in Wireless Sensors," in *IEEE Journal of Solid-State Circuits*, vol. 47, no. 3, pp. 744-756, March 2012.

[9] A. M. R. Dixon, E. G. Allstot, D. Gangopadhyay and D. J. Allstot, "Compressed Sensing System Considerations for ECG and EMG Wireless Biosensors," in *IEEE Transactions on Biomedical Circuits and Systems*, vol. 6, no. 2, pp. 156-166, April 2012.